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PHYSICS OF SPORTS - THE FORWARD PASS

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PHYSICS OF SPORTS - THE FORWARD PASS

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PREFACE

The use of the physics of sports has become increasingly popular in recent years in teaching physics. Sports provide excellent teaching aids to assist the teacher in motivating and maintaining the interest of students. The forward pass in football provides a vehicle for studying many physical phenomena at the intermediate or graduate level. This article uses Lagrange's Equation for a non-conservative holonomic system to develop a mathematical model for the forward pass. The model is used to analyze the motion of the football during flight. Some of the questions answered include: Why does one pass spiral, and another tumble ? What difference does it make whether a quarterback is right-handed or left-handed? Why does a pass tend to nose over on the downward side of the trajectory, but a punt often tends to remain in a fixed direction as the ball travels along the trajectory? The application of physics to the game of football can answer these and many other questions. The study of the forward pass provides the foundation for such complex problems as spin-stabilized satellites, gyroscopes, and complex projectile motion.

PHYSICS OF SPORTS - THE FORWARD PASS

by

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ABSTRACT

The use of the physics of sports has become increasingly popular in recent years in teaching physics. Sports provide excellent teaching aids to assist the teacher in motivating and maintaining the interest of students. The forward pass in football provides a vehicle for studying many physical phenomena at the intermediate or graduate level. This article uses Lagrange's Equation for a non-conservative holonomic system to develop a mathematical model for the forward pass. The model is used to analyze the motion of the football during flight. Some of the questions answered include: Why does one pass spiral, and another tumble ? What difference does it make whether a quarterback is right-handed or left-handed? Why does a pass tend to nose over on the downward side of the trajectory, but a punt often tends to remain in a fixed direction as the ball travels along the trajectory? The application of physics to the game of football can answer these and many other questions. The study of the forward pass provides the foundation for such complex problems as spin-stabilized satellites, gyroscopes, and complex projectile motion.

PHYSICS IN SPORTS--THE FORWARD PASS

It's a beautiful sunny afternoon in Pasadena, and the Rose Bowl is filled to capacity. Pittsburgh and Los Angeles have come on the field for pre-game warmups and the crowd anxiously awaits the start of the Super Bowl. One of the spectators watches Terry Bradshaw throw a beautiful spiralling pass and begins to wonder why the football moves in a given pattern with every throw.

Why does one pass spiral, and another pass tumble? What difference does it make whether a quarterback is right-handed or left-handed? How does the amount of spin on a forward pass affect the distance the pass is thrown? Why does a pass tend to nose over on the downward side of the trajectory, but a punt often tends to remain in a fixed direction as the ball travels along the trajectory? Does a pass remain in one plane when it is thrown? What difference does it make whether the Super Bowl is played in Los Angeles or Pittsburgh, other than familiarity with the home field? Application of physics to the game of football can answer these and other questions. In fact, such questions are priceless vehicles for introducing physics concepts.

A perennial challenge for teachers is to provide instruction in a manner that maintains student interests and stimulates their desires for further knowledge. One recent approach in physics has been to relate physics to sports. For example, the forward pass in football provides an excellent vehicle for studying many physical phenomena. By examining the football in flight as an example of a rigid body in general motion, teachers of physics can lay the foundation for solving such complex and current problems as spin-stabilized satellites, gyroscopes, and complex projectile motion. The football can also be used to study the relation between rotational and translational kinetic energy, the motion of a body through a fluid, the effect of the earth's rotation on a projectile, and numerous other phenomena.

Mathematical Model

To determine the equations of motion for the football during flight, one must determine the forces acting on the football and establish coordinate systems. Three such systems facilitate solution of the problem: the fixed coordinates (x_o, y_o, z_o), the coordinates (X,Y,Z) parallel to (x_o, y_o, z_o) but through the center of mass of the football, o, and coordinates (x, y, z) fixed neither in space nor in the football. The z axis coincides with the axis of symmetry and is, therefore, a principal axis. The y axis is perpendicular to the z axis and in the z-Z plane. The x axis is defined by $\hat{i} = \hat{j} \times \hat{k}$, the x axis is horizontal and perpendicular to the y-z plane. Since x and y always remain in the plane of symmetry of the football, they are also principal axes. The angles (Θ,ϕ,ψ) define the relative orientation between the (x,y,z) and the (X,Y,Z) coordinate systems (see Figure 1).

The aerodynamic force due to the motion of the football through the air and the force due to gravity are the external forces that act on the football in free flight. To simplify the problem, one assumes that the aerodynamic force acts at a point, a distance r, in front of the center of mass called the center of pressure. He assumes, further, that the force is proportional to the velocity squared, but it points in a direction opposite to the velocity vector (See Figure 2). Let $\vec{u}$ be a unit vector in the direction of the velocity vector, then $\vec{F}=-Kv^2\vec{u}$, where K is an experimentally determined constant. If we divide $\vec{F}$ into components, $\vec{F}=F_{x_o}\hat{i}_o+F_{y_o}\hat{j}_o+F_{z_o}\hat{k}_o$. The velocity of the center of mass with respect to the fixed coordinate system is $\vec{V} = \dot{x}_o\hat{i}_o+\dot{y}_o\hat{j}_o+\dot{z}_o\hat{k}_o$. If this value is used for the velocity, since $\vec{u} = \frac{\vec{v}}{v}$, the aerodynamic force is

$$\vec{F}=-K\frac{V^2}{V}(\dot{x}_o\hat{i}_o+\dot{y}_o\hat{j}_o+\dot{z}_o\hat{k}_o)$$

or

$$\vec{F}=-KV\dot{x}_o\hat{i}_o - KV\dot{y}_o\hat{j}_o-KV\dot{z}_o\hat{k}_o \qquad (1)$$

By applying Lagrange's equation for a nonconservative holonomic system, $\frac{d}{dt}(\frac{\partial T}{\partial \dot{q}_i}) - \frac{\partial T}{\partial q_i}=Q_i$, we obtain the (2) equations of motion. To apply this equation, we must determine the kinetic energy T, the generalized forces Q_i, and the generalized coordinates, q_i. The generalized coordinates are (x_o,y_o,z_o) for the translational motion and (θ,ϕ,ψ) for the rotational motion. The kinetic energy is

$$T=\tfrac{1}{2}MV^2+\tfrac{1}{2}(A\Omega_x^2 + A\Omega_y^2 +B\Omega_z^2), \qquad (3)$$

where A and C are the transverse and axial moments of inertia and $\vec{\Omega}=\Omega_x\hat{i}+\Omega_y\hat{j}+\Omega_z\hat{k}$ is the angular velocity of the football. With the coordinate systems defined above, the angular velocity of the (x,y,z) coordinate system is

$$\vec{\omega} = \omega_x\hat{i}+\omega_y\hat{j}+\omega_z\hat{k}$$

or

$$\vec{\omega} = \dot{\theta}\hat{i}+\dot{\phi}\sin\theta\hat{j}+\dot{\phi}\cos\theta\hat{k}. \qquad (4)$$

The angular velocity of the football is

$$\vec{\Omega}= \dot{\theta}\hat{i}+\dot{\phi}\sin\theta\hat{j}+(\dot{\psi}+\dot{\phi}\cos\theta)\hat{k}. \qquad (5)$$

We obtain the velocity. $\vec{V}$, of the center of mass of the football by taking the time derivative of (x_o,y_o,z_o), which are the coordinates of the origin of the (X,Y,Z) coordinate system with respect to the fixed system. Since the origin of the (X,Y,Z) system lies at the center of mass of the football, the velocity of the center of mass, squared, is $V^2=\dot{x}_o^2+\dot{y}_o^2+\dot{z}_o^2$. By substituting the above values for V^2, and $\vec{\Omega}$ into the equation for the kinetic energy, we find that the kinetic energy of the football is

$$T=\tfrac{1}{2}M(\dot{x}_o^2+\dot{y}_o^2+\dot{z}_o^2)+\tfrac{1}{2}\left[A(\dot{\Theta}^2+\dot{\phi}^2\sin^2\Theta)+C(\dot{\psi}+\dot{\phi}\cos\Theta)^2\right]. \quad (6)$$

The potential energy of the football is U=MgZ.

The generalized forces corresponding to the generalized coordinates (x_o,y_o,z_o) are the components of the forces in each of these coordinate directions. Based on the above discussion of the forces, the generalized forces corresponding to (x_o,y_o,z_o) are

$$Q_{x_o}=-Kv\dot{x}_o;$$

$$Q_{y_o}=-Kv\dot{y}_o;$$

$$Q_{z_o}=-Kv\dot{z}_o-\frac{\partial u}{\partial z_o}=-Kv\dot{z}_o-mg. \quad (7)$$

The generalized forces corresponding to the generalized coordinates (Θ, ϕ, ψ) are the torques about the center of mass caused by the fact that the aerodynamic force, $\vec{F}$, acts at the center of pressure, a distance r from the center of mass. We can assume that $\vec{F}$ acts in the plane formed by the velocity vector $\vec{V}$ and the axis of the projectile. This plane is called the plane of yaw. The torque will be perpendicular to the plane formed by the velocity vector and the axis of symmetry. There will be a torque only in the Θ direction if we assume the plane of yaw corresponds to the z-Z plane. Let δ be the angle between the velocity vector and the z axis, then

$$Q_\Theta = \vec{r} \times \vec{F} = rF \sin\delta ,$$
$$Q_\phi = o,$$
$$Q_\psi = o. \qquad (8)$$

Delta is called the angle of yaw (See Figure 3).

After determining generalized forces, coordinates, and kinetic energy, we can employ Lagrange's equation:

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{x}_o}\right) - \frac{\partial T}{\partial x_o} = Q_{x_o} \qquad \frac{d}{dt}(M\dot{x}_o) = F_{x_o} \qquad M\ddot{x}_o = F_{x_o}$$
$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{y}_o}\right) - \frac{\partial T}{\partial y_o} = Q_{y_o} \qquad \frac{d}{dt}(M\dot{y}_o) = F_{x_o} \qquad M\ddot{y}_o = F_{x_o} \qquad (9)$$
$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{z}_o}\right) - \frac{\partial T}{\partial z_o} = Q_{z_o} \qquad \frac{d}{dt}(M\dot{z}_o) = F_{x_o} - Mg \qquad M\ddot{z}_o = F_{x_o} - Mg$$

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{\Theta}}\right) - \frac{\partial T}{\partial \Theta} = Q_\Theta \qquad \frac{d}{dt}(A\dot{\Theta}) - A\dot{\phi}^2 \sin\Theta \cos\Theta + C\dot{\phi}\,(\dot{\psi}+\dot{\phi}\cos\Theta)\sin\Theta = -Kv^2 r \sin\delta$$
$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{\phi}}\right) - \frac{\partial T}{\partial \phi} = Q_\phi \qquad \frac{d}{dt}\left[A\dot{\phi}\sin^2\Theta + C(\dot{\psi}+\dot{\phi}\cos\Theta)\cos\Theta\right] = 0 \qquad (10)$$
$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{\psi}_o}\right) - \frac{\partial T}{\partial \psi} = Q_\psi \qquad \frac{d}{dt}\left[C(\dot{\psi}+\dot{\phi}\cos\Theta)\right] = 0.$$

The first three equations are the translational equations of motion that could be obtained by applying Newton's Second Law. We can make quantitative conclusions about the motion of the football by writing these equations in terms of the speed V and the angles Θ and ϕ . The velocity components (see Figure 4) along the (X,Y,Z) coordinates are

$$\dot{x}_o = V \sin \Theta_o \cos \phi,$$
$$\dot{y}_o = V \sin \Theta_o \sin \phi,$$
$$\dot{z}_o = V \cos \Theta_o . \tag{11}$$

Differentiation of these equations, results in the following equations,

$$\ddot{x}_o = - V \sin \Theta_o \sin\phi\dot{\phi} + V \cos \phi \cos \Theta_o\dot{\Theta}_o + \dot{V} \sin\Theta_o \cos\phi ,$$
$$\ddot{y}_o = V \sin \Theta_o \cos\phi\dot{\phi} + V \sin \phi \cos \Theta_o\dot{\Theta}_o + \dot{V} \sin\Theta_o \sin \phi,$$
$$\ddot{z}_o = - V \sin\Theta_o \dot{\Theta}_o + \dot{V} \cos \Theta_o . \tag{12}$$

When we substitute equations (11) and (12) into (9), we have

$$M(-V \sin \Theta_o \sin\phi\dot{\phi} +V \cos\phi \cos \Theta_o \dot{\Theta}_o+\dot{V} \sin \Theta_o \cos \phi)=-KV^2\sin\Theta_o\cos \phi,$$
$$M(V \sin \Theta_o\cos \phi\dot{\phi}+ V \sin\phi\cos \Theta_o \dot{\Theta}_o+ \dot{V} \sin \Theta_o\sin \phi) =- KV^2 \sin\Theta_o \sin \phi, \tag{13}$$
$$M(-V \sin \Theta_o \dot{\Theta}_o + \dot{V} \cos \Theta_o) =-KV^2 \cos \Theta_o - mg .$$

The solution of the first equation for $\dot{\phi}_o$ yields

$$\dot{\phi}_o = \frac{MV \cos\phi \cos\Theta_o \dot{\Theta}_o+M\dot{V} \sin \Theta_o\cos\phi + KV^2 \sin\Theta_o \cos \phi_o}{MV \sin \Theta_o \sin \phi} . \tag{14}$$

By substituting this value for $\dot{\phi}$ into the second equation and simplifying, we obtain the equation

$$MV \cos^2\phi \cos \Theta_o \dot{\Theta}_o + M\dot{V} \sin \Theta_o \cos^2\phi + KV^2 \sin \Theta_o \cos^2\phi$$
$$+MV \sin^2\phi \cos \Theta_o \dot{\Theta}_o + M\dot{V} \sin\Theta_o \sin^2\phi =-KV^2 \sin \Theta_o \sin^2\phi$$

or

$$MV \cos \theta_o \dot{\theta}_o + M\dot{V} \sin \theta_o + KV^2 \sin \theta_o = 0. \quad (15)$$

The solution of the third equation for $\dot{\theta}_o$ yields

$$\dot{\theta}_o = \frac{KV^2 \cos \theta_o + mg + M\dot{V} \cos \theta_o}{MV \sin \theta_o} . \quad (16)$$

By substituting $\dot{\theta}_o$ into (15) and simplifying, we have

$$\dot{V} = -\frac{KV^2}{M} - g \cos \theta_o . \quad (17)$$

The equation for θ_o may be simplified by substituting this value for $\dot{V}$ into (16) and simplifying. The result is

$$\dot{\theta}_o = \frac{g \sin \theta_o}{V}. \quad (18)$$

Using these values for θ_o and $\dot{V}$ in equation (14), yields

$$\dot{\phi} = 0. \quad (19)$$

McCusky developed these equations for a spin stabilized projectile.[1] As he noted, the second equation indicated that $\dot{\theta}_o$ is always positive since θ_o lies between zero and pi. Thus, the time rate of change of θ_o is positive; i.e., θ_o increases with time. The trajectory is concaved downward. The last equation implies that ϕ is a constant; therefore, the trajectory is a plane curve. This result is based on the simplifying assumptions made in formulating the model. (We later discuss forces that cause the football to move out of its original plane.)

The last three equations obtained by applying Lagrange's equation are the rotational equations of motion that could be obtained by applying the rotational analog to Newton's Second Law; i.e., the torque is equal to the time rate of change of the angular momentum, $N=\frac{d\vec{L}}{dt}$. We can determine the requirement for the football to remain stable from these equations. Stability refers to the characteristic that allows the football to maintain colinearity between its axis of symmetry and the tangent to the trajectory at the center of mass. This is equivalent to the axis of symmetry and the velocity vector pointing in the same direction. In this case, the angle $\Theta=\Theta_o$ and $N_\Theta=o$. The rotational equations of motion reduce to

$$A\ddot{\Theta}-A\dot{\phi}^2 \sin\Theta\cos\Theta+C\dot{\phi}\, s\sin\Theta = o,$$

$$\frac{d}{dt}\left[A\dot{\phi}\sin^2\Theta+Cs\cos\Theta\right]=o, \qquad (20)$$

$$\frac{d}{dt}\left[Cs\right]=o,$$

where $s=\dot{\psi}+\dot{\phi}\cos\Theta$ is the spin of the football about the z axis. By integrating the second and third equation, we have

$$A\dot{\phi}\sin^2\Theta+Cs\cos\Theta=\alpha\;,$$

$$Cs=\beta, \qquad (21)$$

where α and β are constants. The first of these equations may be solved for

$$\dot{\phi} = \frac{\alpha - Cs\cos\theta}{A\sin^2\theta}. \tag{22}$$

To determine the conditions under which the football remains stable, we assume that it is disturbed slightly and find the conditions under which it returns to, or oscillates about, the equilibrium position. We let $\theta - \theta_1 = \delta$ represent a small angular deviation of the axis of symmetry from the velocity vector, and we let η represent the corresponding perturbation of the angle ϕ i.e., $\eta = \phi - \phi_1$. (Note that the subscript 1 denotes the equilibrium or unperturbed value.) We desire the motion about this equilibrium position; thus, we treat these values as constant quantities. Then substituting $\theta = \delta + \theta_1$ and $\phi = \eta + \phi_1$ into the first of the rotational equations of motion (Equation 20), we have

$$rF\sin\delta = A\ddot{\delta} + Cs\dot{\eta}\sin(\delta+\theta_1) - A\dot{\eta}^2\cos(\delta+\theta_1)\sin(\delta+\theta_1). \tag{23}$$

since $\cos(\delta+\theta_1) = \cos\theta_1 - \delta\sin\theta_1$ and $\sin(\delta+\theta_1) = \delta\cos\theta_1 + \sin\theta_1$,

$$rF\sin\delta = A\ddot{\delta} + Cs\dot{\eta}(\delta\cos\theta_1 + \sin\theta_1) - A\dot{\eta}^2(\cos\theta_1 - \delta\sin\theta_1)(\delta\cos\theta_1 + \sin\theta_1) \tag{24}$$

or

$$rF\sin\delta = A\ddot{\delta} + Cs\dot{\eta}(\delta\cos\theta_1 + \sin\theta_1) - A\dot{\eta}^2(\cos\theta_1\sin\theta_1 - \sin^2\theta_1 + \delta\cos^2\theta_1) \tag{25}$$

By considering only first order terms in δ and $\dot{\eta}$ and considering δ small, we have

$$rF \sin \delta = A\ddot{\delta} + Cs\dot{\eta} \sin\theta_1 \tag{26}$$

or

$$\ddot{\delta} + \frac{Cs \sin\theta_1}{A} \dot{\eta} - \frac{rF}{A} \delta = o. \tag{27}$$

From equation (22), we have

$$\phi_1 = \frac{\alpha - Cs \cos \theta_1}{A \sin^2 \theta_1} \quad . \tag{28}$$

Since we take the motion about the equilibrium position, we treat ϕ_1 and θ_1 as constants, $\dot{\phi}_1 = o$ and $\alpha = Cs \cos \theta_1$. By using this value for α and substituting $\phi = \eta + \phi_1$ and $\theta = \delta + \theta_1$ into equation (21), we have

$$\dot{\eta} = \frac{Cs \cos \theta_1 - Cs \cos (\delta + \theta_1)}{A \sin^2 (\delta + \theta_1)} \tag{29}$$

or

$$\dot{\eta} = \frac{Cs \cos \theta_1 - Cs(\cos \theta_1 - \delta \sin \theta_1)}{A(\delta\cos^2\theta_1 + 2\,\delta\cos \theta_1 \sin \theta_1 + \sin^2\theta_1)}. \tag{30}$$

By neglecting higher ordered terms, we have

$$\dot{\eta} = \frac{Cs\delta \sin \theta_1}{A \sin^2 \theta_1} = \frac{Cs\delta}{A \sin \theta_1} . \tag{31}$$

Substituting equation (31) into equation (27),

$$\ddot{\delta} + \left[\frac{(Cs)^2}{A^2} - \frac{rF}{A}\right] \delta = o. \tag{32}$$

For δ to be periodic

$$\frac{(Cs)^2}{A^2} - \frac{rF}{A} > o. \tag{33}$$

Therefore, the football remains stable if

$$s^2 > \frac{rFA}{C^2} . \tag{34}$$

Thus, if a pass has a spin less than the spin required by this inequality, it will tumble. If the spin satisfies this inequality, the football will remain stable.

We can obtain information about the orientation of the football by continuing the analysis of the rotational equations of motion. Returning to equation (31), we have

$$\dot{\eta} = \frac{Cs\delta}{A \sin\Theta_1} .$$

Since $\dot{\phi} = o$, the spin of the football, s, is just $\dot{\psi}$. Thus,

$$\dot{\eta} = \frac{C\dot{\psi}\delta}{A \sin \Theta_1} .$$

Unless the spin of the football is excessive, $\dot{\eta}$ will be small when $\dot{\delta}$ is small. In other words, the perturbation of the angular velocity about the vertical will be small as long as the axis of spin differs from the tangent to the path by only a small amount. Note if $\delta > o$, the nose of the football dips below the tangent to the trajcctory, $\dot{\eta} > o$, for positive spin, and the football has a tendency to turn to the left as seen by the quarterback. If $\delta < o$, the nose points upward with respect to the tangent to the path, $\dot{\eta} < o$, and the football has a tendency to turn to the right. By using the definition for positive spin, we find that a right-handed quarterback, such as Roger Staubach, would impart a positive spin to the football, and a left-handed quarterback, such as Ken Stabler, would impart a negative spin.

Additional Physical Phenomena

The football provides the vehicle for studying numerous other physical phenomena. Many of these phenomena make the problem more complex than it appears in the model used thus far. For example, Roger Staubach has developed his talent as a quarterback in much the same way that an experimental physicist or engineer develops a new device. He understood the basic principles of throwing the football. When he first threw the football, it may have slipped out of his hand and tumbled in the wrong direction. Through experimentation, he learned the best grip to use in throwing the football, but it still had the tumbling motion. Through further experimentation and practice, he learned that, by spinning the ball, he could stop the tumbling motion and throw the perfect spiral.

Just how much spin does the quarterback need to give the football? Is there an optimum spin? Is it possible to give too much spin? From the mathematical model we developed, we saw that a minimum spin is required for the football to remain stable, $s^2 > \frac{rFA}{C^2}$. When the quarterback throws the football, he gives it an initial amount of kinetic energy. This kinetic energy can be divided into two parts - translational kinetic energy, $\frac{1}{2}MV^2$, and rotational kinetic energy, $\frac{1}{2}(A\Omega_x^2+A\Omega_y^2+C\Omega_z^2)$. (Thus $T_i=\frac{1}{2}MV_i+\frac{1}{2}(A\Omega_x^2+A\Omega_y^2+C\Omega_z^2)$. For a given value of the initial kinetic energy, an increase in spin of the football must result in a decrease in the initial velocity of the football. From elementary projectile motion, a decrease in the initial velocity for a fixed angle of release will result in a shorter pass. If the quarterback expects to throw a spiralling pass that will cover the maximum distance, he must achieve a delicate balance. He must provide enough spin for stability, but not an excessive amount that will result in decreased distance.

Excessive spin can cause other problems in addition to reduced distance. Gyroscopes work on the principle that a rotating body resists change in the direction of its axis of rotation. If a rigid body with an axis of symmetry rotates fast enough, the axis of rotation will tend to point in a fixed direction. If the football is given an excessive spin, it tends to act as a gyroscope. The axis of rotation remains pointed in a fixed direction in space (see Figure 5). The football appears to "float," and it fails to nose over on the downward side of the trajectory. The quarterback seldom sees this effect when he throws the ball, since he rarely imparts enough rotation with his hand. However, a punter frequently sees it when he kicks the football.

Three basic perturbing effects act on the football - the gyroscopic effect, the Poisson effect, and the Magnus effect. In most cases ~~,~~ the gyroscopic effect is predominant. A detailed mathematical discussion of the gyroscopic effect can be developed from the mathematical model previously formulated.

The basic equation is $\vec{N} = \frac{d\vec{L}}{dt}$: the torque is equal to the time rate of change of the angular momentum. When the spin and thus the angular momentum is large enough, the football becomes gyrostabilized. We saw the same effect in our discussion of the football with excessive spin. If an external torque acts on the football, the spin axis changes direction. This change of direction of the spin axis is known as precession. The football precesses at a rate proportional to the amount of torque applied and precesses about an axis perpendicular to the torque. The torque on the football is caused by the aerodynamic force, $\vec{F}$, acting at the center of pressure, a distance r from the center of mass. This results in a torque, $\vec{N} = \vec{r} \times \vec{F}$. We now assume that $\vec{F}$ acts in the plane of yaw. If the football has a right-hand spin, the angular momentum $\vec{L}$ is directed along the symmetry axis. The symmetry axis will precess about the velocity vector (tangent to the trajectory) in such a way that the nose of the football moves to the right when it is viewed from the rear. As soon as the nose of the football leaves the original vertical plane, a component of the aerodynamic force acts to the right and tends to push the football to the right out of the original vertical plane.

Experiments with spherical projectiles have shown that Magnus effects predominate over Poisson effects. We first consider the Magnus effects. If a rotating cylinder is placed in a uniform stream, there is a resultant aerodynamic force on the cylinder. The rotation of the cylinder tends to retard the flow at the bottom and speed the flow at the top of the cylinder. This results in increased pressure at A and decreased pressure at B, as shown in Figure 6. The pressure differential produces a force that acts from A to B. If a rotating football is placed in a uniform stream, as shown in Figure 6, the velocity of the stream can be resolved into components along the symmetry axis and perpendicular to it. The velocity component perpendicular to the axis produces a force as discussed with the rotating cylinder. If the football has right-handed spin, the force will point toward the left and cause a deviation to the left out of the original plane.

We now consider the Poisson effect. When there is an angle of yaw, a cushion of compressed air is developed in front of the football. Since the football is spinning, it has a tendency to roll over this cushion of air as a cylinder rolls on a rough surface. If it is given a right-hand spin, the football will roll to the right of the original plane.

Our discussion of these basic perturbing effects shows that forces tend to push the football out of its original plane. Simplifying assumptions have eliminated other complicating conditions. In the case of the football, not only the magnitude and direction but also the point of application of the aerodynamic force changes. A right-hand spin causes the nose of the football to follow an oscillating line that remains mainly to the right of the vertical plane through the tangent to the trajectory.[2]

The final question raised in the introduction was this: what difference does it make whether the Steelers play the Rams in Los Angeles or Pittsburgh? In all previous discussions we have neglected the rotation of the earth and the effects of air density on the football.

Both of these conditions affect the flight of the football. Most textbooks on dynamics discuss the earth's rotation on a projectile in flight. The magnitude of the effect depends on the latitude. The closer the projectile to the equator; the smaller the force. In the northern hemisphere, projectiles tend to drift to the right because of this corriolis force. Thus, the drift caused by corriolis would be less in Los Angeles than in Pittsburgh. Because of the short distances a football is thrown, the effect would be very small in either location. If the game were played in Denver, the lower air density in Denver would result in slightly longer passes.

In studying the forward pass in football we have discussed many basic physics principles and have discovered some interesting facts about football. A pass spirals because of the magnitude of its spin. Since a right-handed quarterback gives the football a positive spin, the football tends to turn slightly to the right and drift slightly to the right out of the original vertical plane. The opposite is true for a left-handed quarterback. The quarterback must maintain a delicate balance, he must achieve enough spin for stability, but too much spin will reduce the distance of the pass. And, if he gives a football too much spin, which often occurs in punts, it fails to nose over on the downward side of the trajectory. The axis of symmetry remains in a fixed direction, and the football is gyrostabilized.

The use of the physics of sports has given the physics teacher a valuable teaching aid to maintain the interests of students and motivate them toward additional study. The author has only touched the surface in discussing the physical phenomena in the latter part of the article. A much more detailed discussion could be accomplished in the classroom. Teachers should also encourage their students to look into such problems. Why? In order for them to integrate the basic studies of physics they have completed. The forward pass vividly demonstrates the value of an interdisciplinary background. The problems students will face in tomorrow's world dictate the need for at least a fundamental understanding of all basic physics disciplines.

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FOOTNOTES

[1]McCusky, S. W., An Introduction to Advanced Dynamics (Addison-Wesley Publishing Company, Reading, Massachusetts, 1959), pp. 139-141.

[2]Kooy, J. M. and Uytenbogaart, J. W. H., Ballistics of the Future (McGraw-Hill, New York, 1946), pp. 181-182.

CAPTIONS

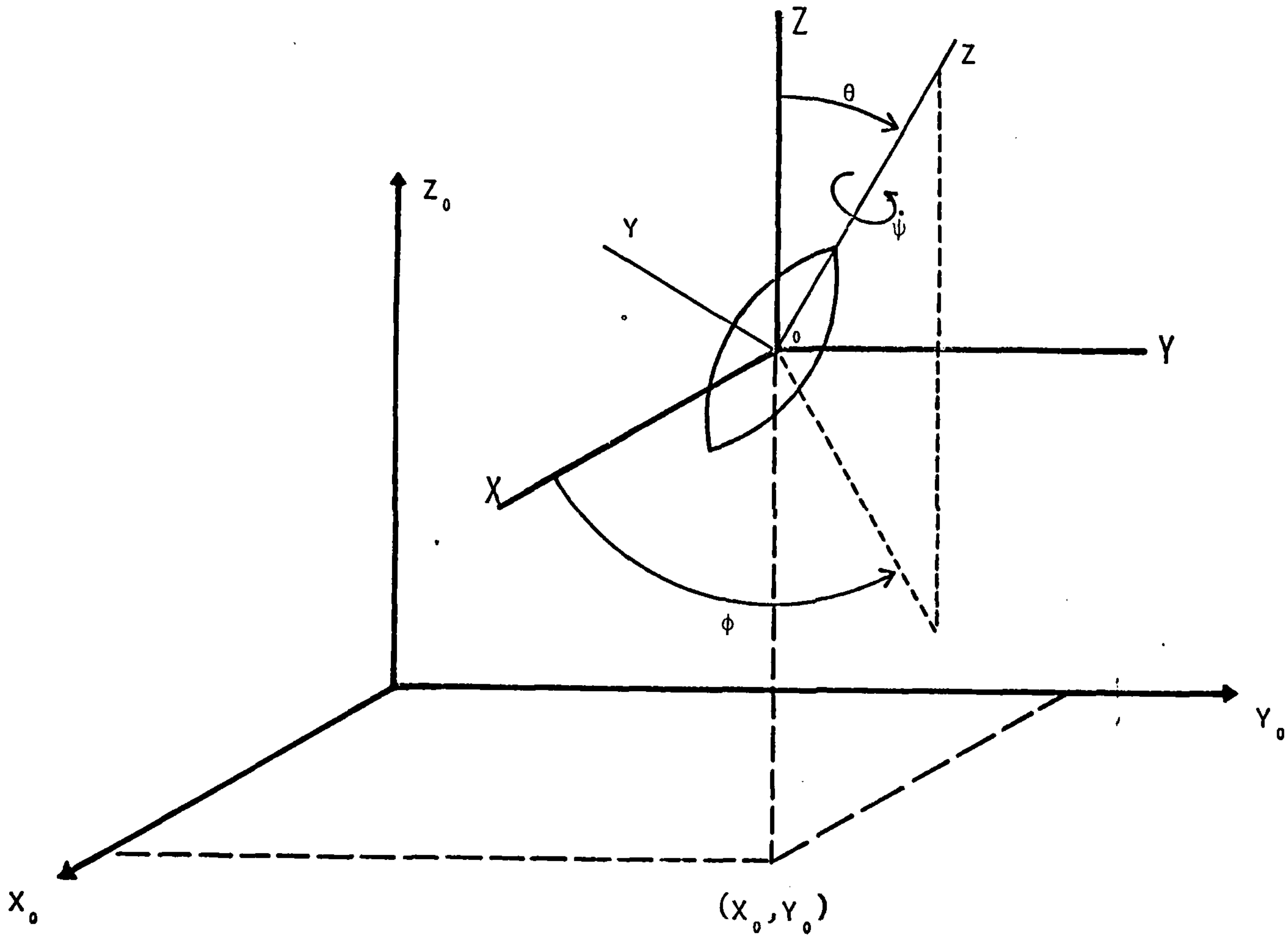

FIGURE 1: COORDINATE SYSTEMS MICHAEL J. [illegible]

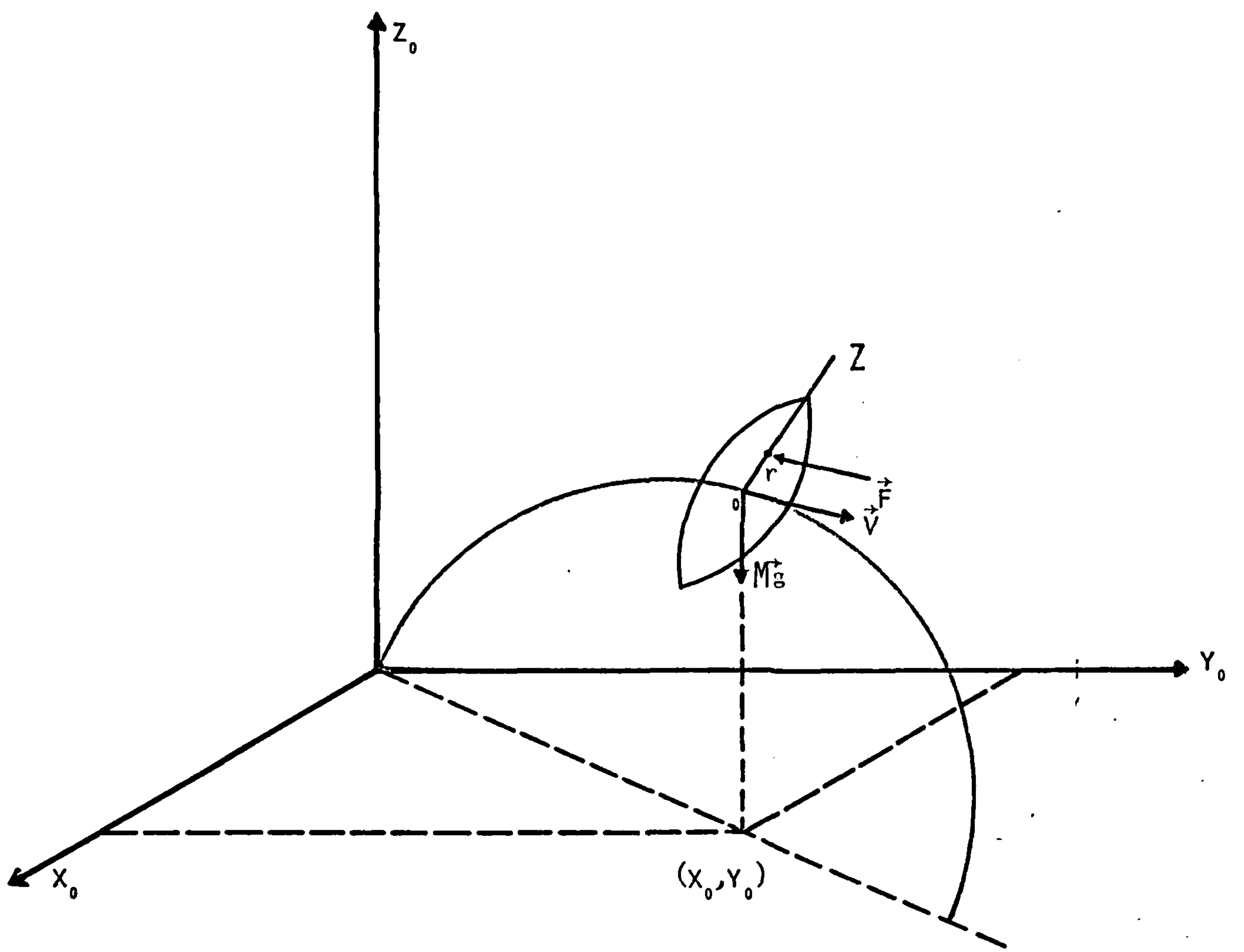

FIGURE 2 · FORCES ON THE FOOTBALL MICHAEL TOOLE CAM 100% FOR 8½ X 11

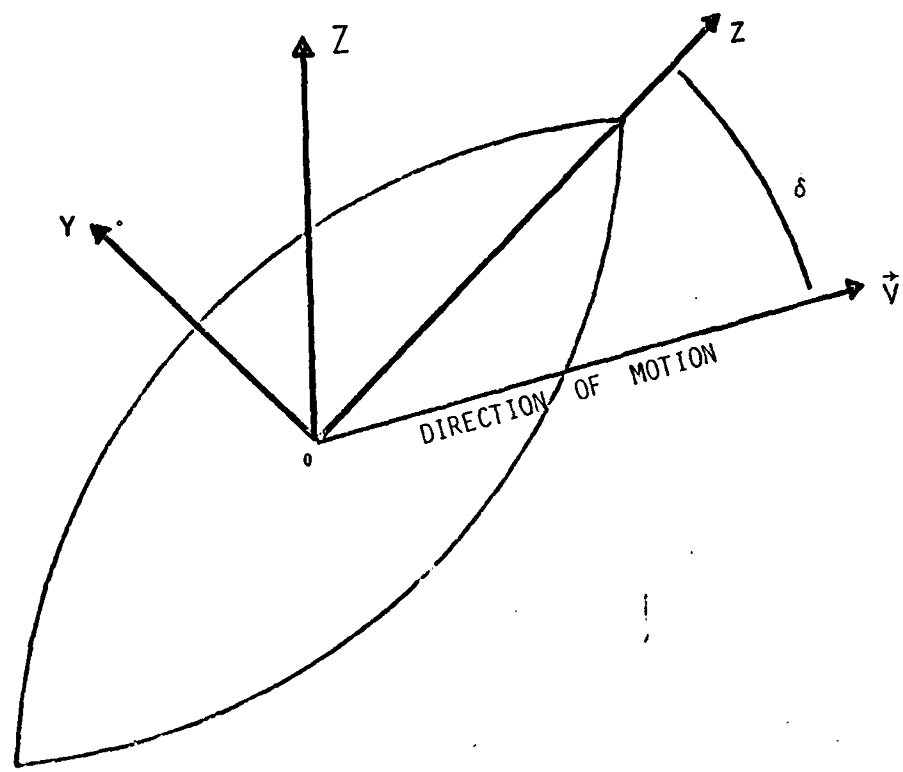

FIGURE [illegible]. PLANE OF YAW AND ANGLE OF YAW

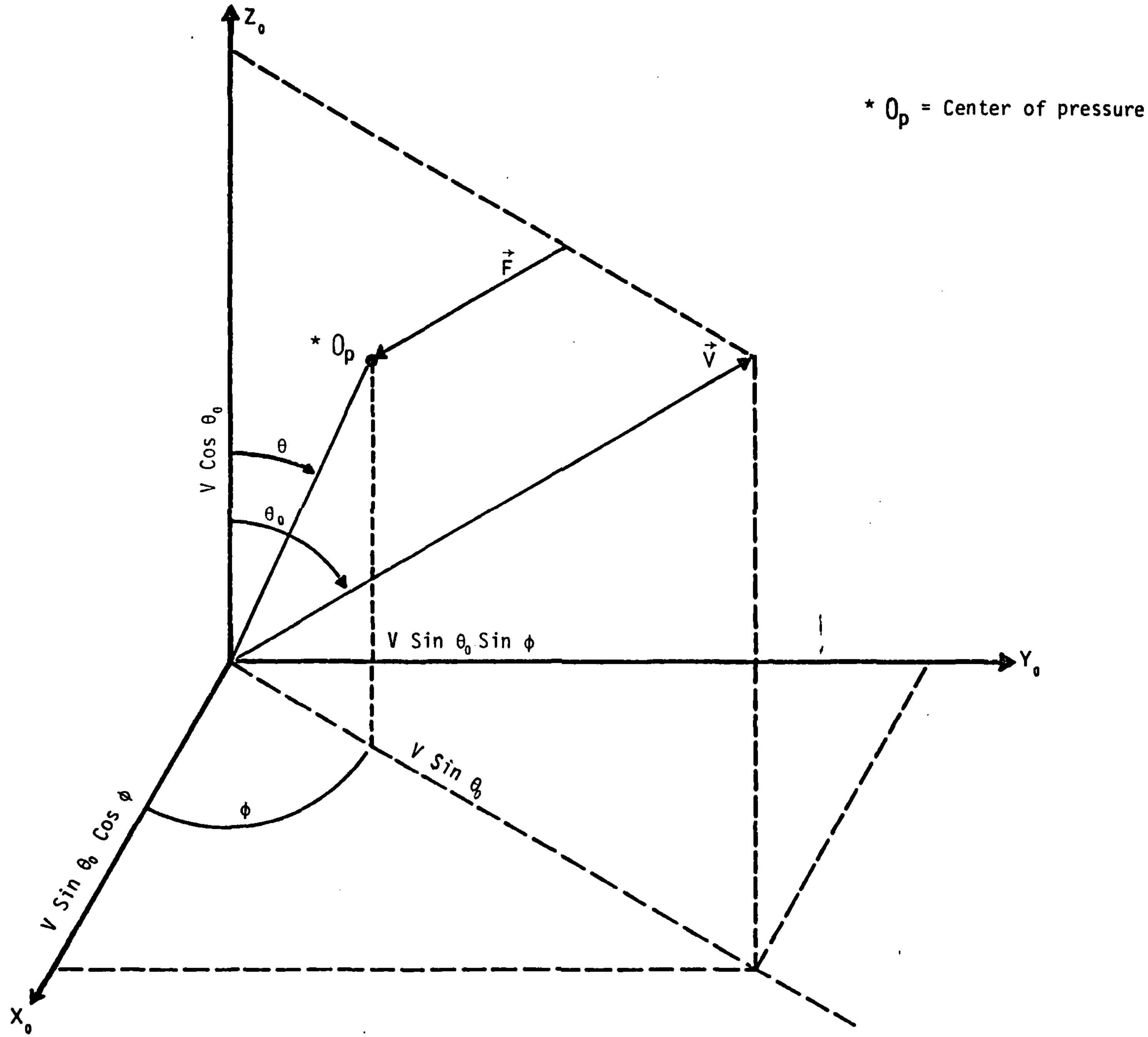

FIGURE 4: VELOCITY COMPONENTS MICHAEL TAKE CAM 100% FOR 8½X11

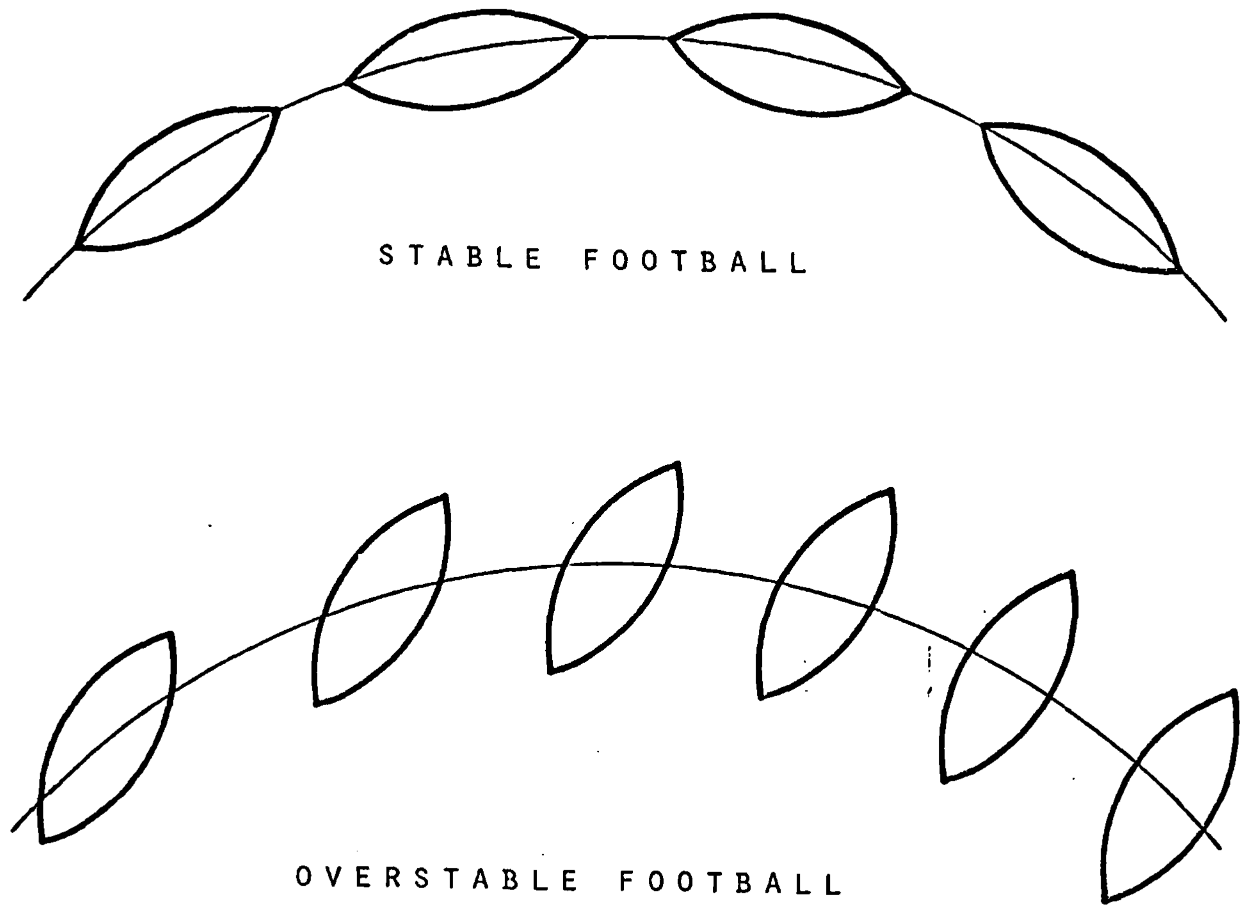

FIGURE 5: STABLE AND OVERSTABLE FOOTBALLS MICHAEL TOOLE CAM 100% FOR 8½ X 11

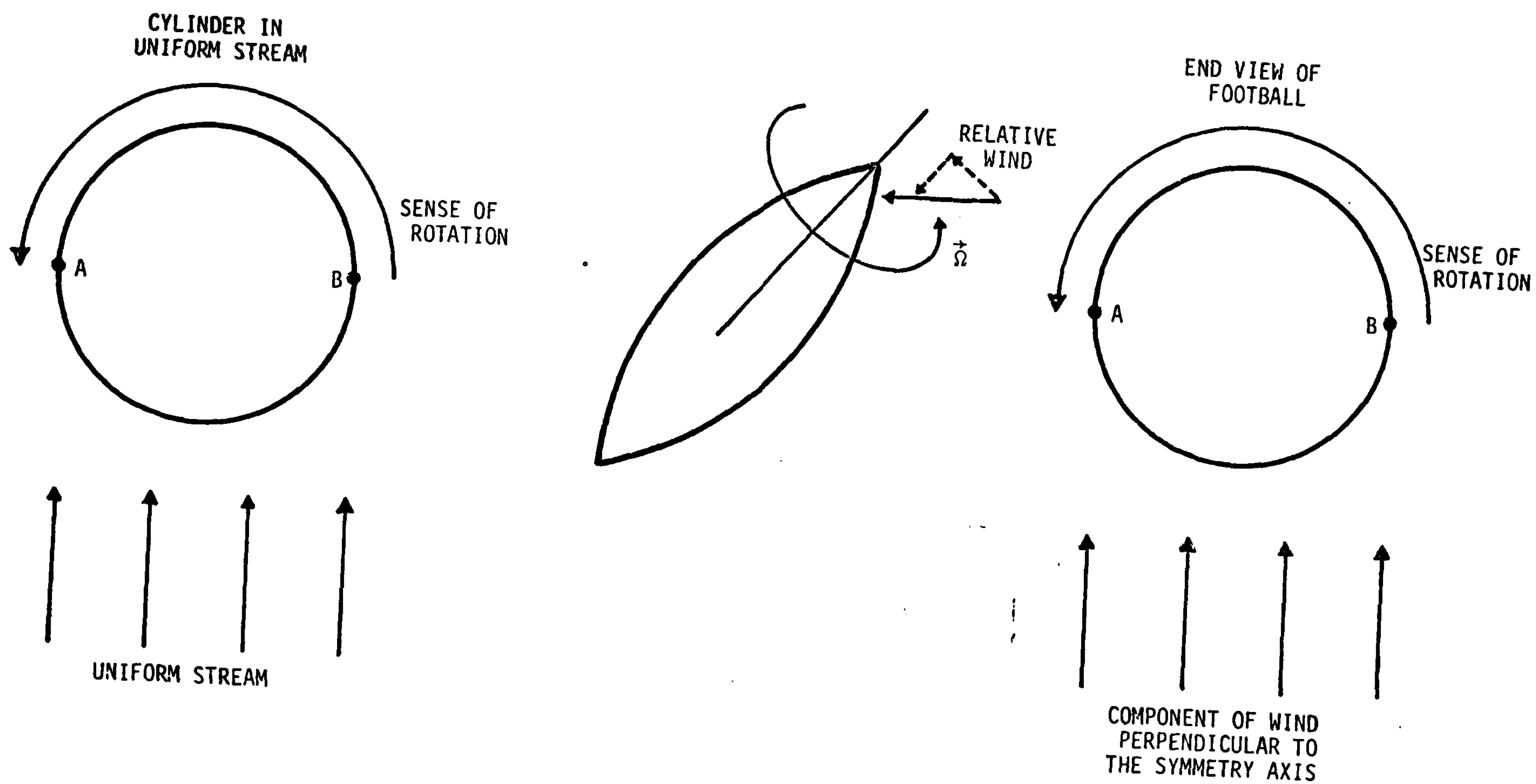

FIGURE 6: MAGNUS EFFECT MICHAEL TOOLE

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